Chapter 6 Waves

- This chapter discusses waves and illustrates the wave properties of light
- Very important in today's technology:
  - Make measurement to 1/1000,000 of a meter (high precision manufacture)
  - Design antireflective coating and laser oscillators
- Also deal with the question of the energy of light signal

Outline

- Periodic motion
- Energy in a wave
- The principle of superposition
- Coherence
- Young's experiment
- Polarization

Periodic Motion

- In case of water waves the water itself moves vertically while the wave progresses horizontally.
- Fig. 6.1: several different forms of waves.
- ***Fourier analysis*** – All other waves shown in Figs. 6.1(a)-6.1(c) can be represented as a sum of sinusoidal waves shown in Fig. 6.1(d).

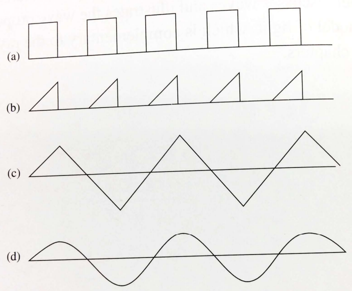

FIGURE 6.1. Four different waveforms. All are characterized by a repetitious pattern.

- Waves are a function of both position and time
- The dependent variable here is the displacement of the medium ***up and down*** at right angles to the direction of propagation of the wave – ***transverse*** waves
  - For example – sound waves
- ***Longitudinal*** waves – the motion of the medium is ***back and forth*** to the direction of propagation.

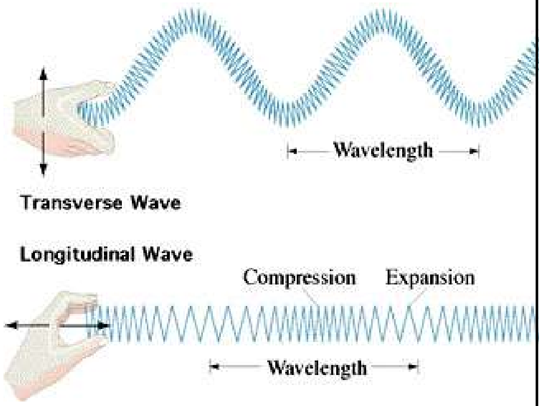

Figure 6.2

- The parameters used to describe a sinusoidal wave
  - Amplitude *A* – the maximum displacement of the wave from its equilibrium position along the x-axis
  - Wavelength λ, the distance between two similar points along the wave train
- Expression for the wave:

  (spatial dependence of wave)

$$f(x) = A \sin 2\pi \frac{x}{\lambda}$$

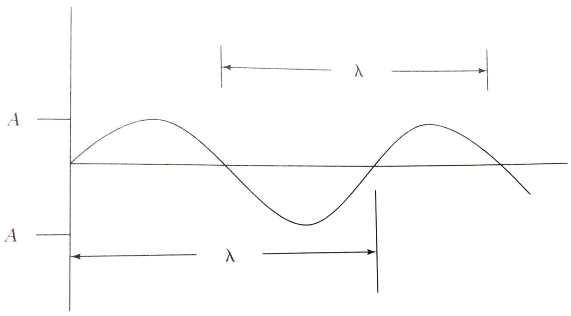

FIGURE 6.2. A sinusoidal wave showing the amplitude *A* and the wavelength λ.

Figure 6.3: Time dependence of wave

- Line *LL'* – procession of the wave over successive time intervals
- Period *T*, the time required for the wave to move through one cycle
- Frequency *f*, the number of times a maximum is reached at some fixed point per unit time (hertz, Hz)

$$f = \frac{1}{T}$$

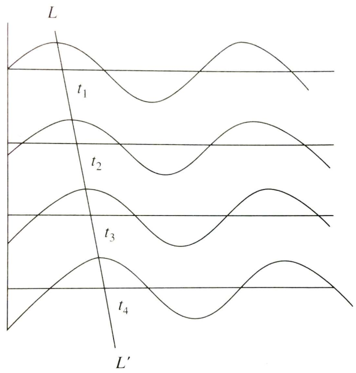

FIGURE 6.3. The sinusoidal wave at four successive points in time. The line *LL'* illustrates how the peak of the wave has moved over the time period from t_1 to t_4.

- Angular frequency in radians per second is given by $\omega = 2\pi f$
- The time dependence is given by

$$f(t) = A\sin 2\pi ft = A\sin \omega t$$

- Combine Eqs. (6.1) and (6.3), the expression for the wave as a function of *x* and *t*,

$$f(x,t) = A\sin 2\pi(\frac{x}{\lambda} - ft)$$

- If the wave is propagating in the ***positive*** *x*-direction, the sign of the spatial and time parts of the argument will be different. Otherwise, the wave propagates in the ***negative*** *x*-direction.

Example 6.1

- Write the equation for a wave with amplitude 3 cm, wavelength 5 cm, and frequency 30 Hz. What are the period and the angular frequency of this wave?

Phase: angle of the wave $2\pi(\frac{x}{\lambda} - ft)$

- Vary the phase by simply adding a constant and become

$$2\pi(\frac{x}{\lambda} - ft + \Phi)$$

- Using the definitions of frequency and wavelength, we can find the velocity of wave. In one period the wave advances one wavelength, so the velocity of the wave v is

$$v = \frac{\lambda}{T} = f\lambda$$

Example 6.2

- What is the range of the frequencies of visible light?

Example 6.3

A particular wave is represented by the expression

$$y = 10\sin(1.2566\times10^{7}x + 3.740\times10^{15}t + \frac{\pi}{2})$$

What are the amplitude, wavelength, frequency, velocity, and phase of the wave?

Energy in a Wave

- How much energy passes some fixed point in space during some time interval t?
- More commonly we refer to the ***intensity*** of wave, the power crossing a unit area in unit time.
- Intensity of light wave: what is measured in evaluating the illumination generated by a light source, and is ***proportional to the square of the amplitude*** of wave

The Principle of Superposition

- Arise when two waves pass the same point in space simultaneously
- The instantaneous displacement of the medium is the sum of the instantaneous displacements of each of the component waves.
- Fig. 6.4 – when two waves with the same frequency and in phase are superposed.

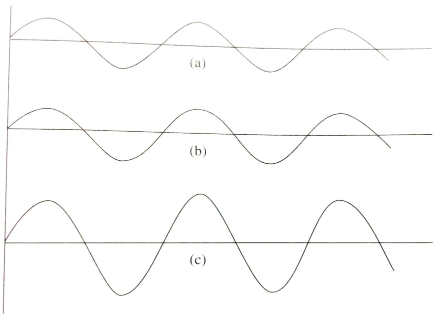

FIGURE 6.4. Two waves (a) and (b) with the same frequency, amplitude, and phase when superposed give wave (c) with the same frequency and phase but with twice the amplitude.

Figure 6.5

- In contrast, two waves with the same frequency and amplitude but with a 180° phase difference.

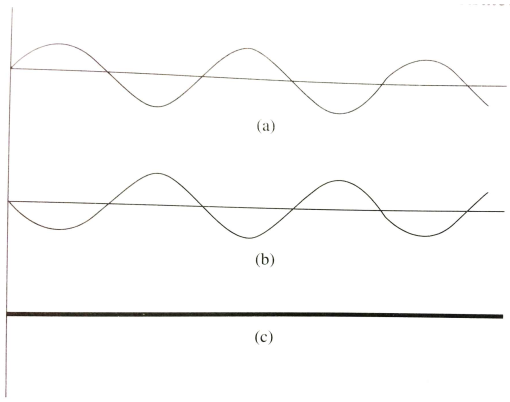

FIGURE 6.5. Waves (a) and (b) differ only in phase, in this case by π radians, but in superposition the disturbances cancel each other.

- There are other situations in which the phase is in some intermediate state or the waves have different wavelengths.
- Two waves differ only in phase:

$$y = A_1 \sin 2\pi(\frac{x}{\lambda} - ft + \Phi_1) + A_2 \sin 2\pi(\frac{x}{\lambda} - ft + \Phi_2)$$

$$y = B \sin 2\pi(\frac{x}{\lambda} - ft + \Phi), \text{with}$$

$$B = \frac{A_1 \sin\phi_1 + A_2 \sin\phi_2}{\sin\Phi} \quad \text{and} \quad \Phi = \tan^{-1}\left[\frac{A_1 \sin\phi_1 + A_2 \sin\phi_2}{A_1 \cos\phi_1 + A_2 \cos\phi_2}\right].$$

Example 6.4

- Find the amplitude and phase of the sum of the waves

$$y_1 = 7\sin 2\pi(\frac{x}{4} - 20t + 30^o)$$

$$y_2 = 4\sin 2\pi(\frac{x}{4} - 20t + 45^o)$$

Coherence

- Typically we deal with the superposition only of waves of identical frequency.
- Two light bulbs in a lamp do not demonstrate a superposition – there are no bright and dark regions obtained.
- Light arises as a result of an atomic process and is emitted from a source in short pulses know as ***photons***. Typically, photons can superpose only with themselves.
- When two sources or a single source divided into two paths have a fixed phase relationship, so that Φ_1 and Φ_2 are known relatively, the sources are said to be ***coherent***.

Young's Experiment – Figure 6.6

- The distribution of ligh passing through two slits in a screen.
- At the screen, two beams superposed and gave rise to a series of bright and dark bands known as ***fringes***.
- This process, the superposition of light waves, is known as ***interference***.

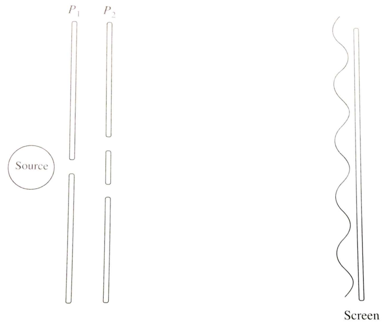

FIGURE 6.6. Young's experiment. Pinhole P_1 limited the source and established a coherent output. Slits P_2 divided the output into two beams that then interfered and gave the intensity distribution on the screen illustrated by the curve.

Figure 6.7 – How these fringes arise?

- The paths from the two slits S_1 and S_2, the distance between the slits d, and the distance to the screen D
- Concern with the light at point y above the centerline of the screen.
- The intensity at centerline is a maximum because → equidistant from two slits → the waves from each slit are in phase

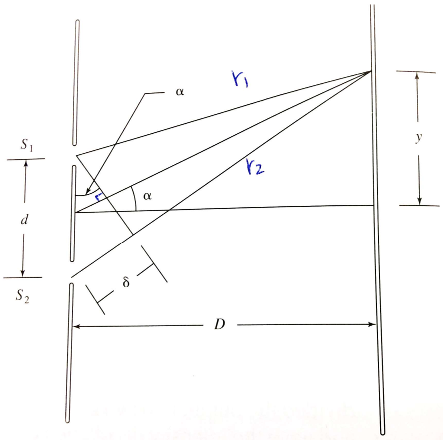

FIGURE 6.7. The geometry of Young's experiment.

- If α is small enough, then $\sin\alpha \approx \tan\alpha.$
- The distance to the point *y* above the centerline δ is $\frac{\delta}{d} = \frac{y}{D}$ and $\delta = \frac{dy}{D}.$
- If δ is an even multiple of wavelengths, the display on the screen at *y* will be a maximum

$$n\lambda = \frac{dy}{D}$$

and if δ is an odd multiple of half-wavelength, the display will be a minimum

$$(2n+1)\lambda = \frac{dy}{D},\ n \text{ is an integer, } n = 0,1,2,3,....$$

Example 6.5

- How far from the centerline in a Young's experiment is the third maximum? The wavelength of the source is 550-nm green light, the screen is 1 m from the slits, and the slits are 15 um apart.

Polarization

- A property of transverse waves not shared by longitudinal waves is ***polarization***.
- The amplitude vector of the transverse electromagnetic wave is ***perpendicular*** to the direction of propagation.
- Fig. 6.8: The *x*-*z* plane is called the ***plane of polarization***.
- If the displacement, $D_{x,z}$ remains in the *x*-*z* plane, the wave is said to be ***plane polarized***.
- If *D* rotates about the direction of propagation *z*, the wave is said to be ***circularly polarized*** or ***elliptically polarized*** if *D* varies from *x*-*z* to *y*-*z* planes as *D* rotates.

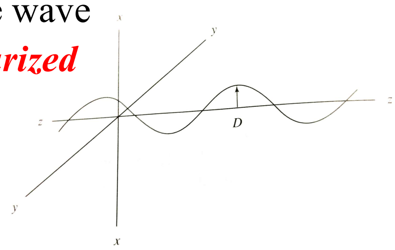

FIGURE 6.8. A wave plane polarized in the x-z plane.

- Most light sources are randomly polarized, emitting light with all planes of polarization.
- To achieve a polarized wave from the randomly polarized source, the simplest approach uses a filter that selects one polarization plane.
  - Use ***dichroic*** material
  - Allowing only those wave components polarized parallel to its axis to pass through
- If the axes of the sheets of two filters are parallel, no change in intensity occurs; however, if the axes are perpendicular, no light emerges from the second sheet.

Figure 6.9

- The first sheet is called the ***polarizer***, it has the effect of fixing the plane of polarization of the beam.
- A second filter call ***analyzer*** is used to establish whether the beam is polarized.
- If the angle between the plane of polarization and the analyzer *A* is θ, then the intensity transmitted through *A* is given by

$$I_A = I_P \cos^2 \theta,$$

where I_A is the intensity after the beam passing the analyzer, and I_P is the intensity of the wave incident on the analyzer.

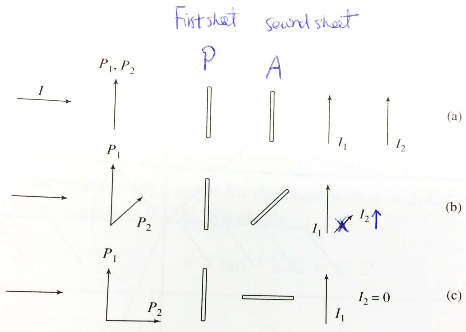

FIGURE 6.9. Polarization of light of intensity I as it passes through Polaroid sheets P_1, P_2. (a) The polarization axes of sheets 1 and 2 parallel. I_1 and I_2 are the relative intensities after the beam has passed through each respectively. (b) P_1 and P_2 set obliquely. (c) P_1 and P_2 set at right angles to each other.

Example 6.6

- The angle between the plane of polarization of a wave and an analyzer is 40°. What is the intensity I_A of the wave after passing through the analyzer?

Fig. 6.10

- Light falling on a ***nonmetallic*** surface is separated into reflected and refracted waves.
- The reflected ray is ***partially*** polarized, with its polarization ***parallel*** to the reflecting surface.
- When the angle θ between the reflected and refracted waves is 90°, the reflected wave is found to be ***fully plane polarized***.
- The condition is met when $n = \tan i$, where n is the refraction index of the medium and i is the angle of incidence.
- This expression is known as ***Brewster's law***.

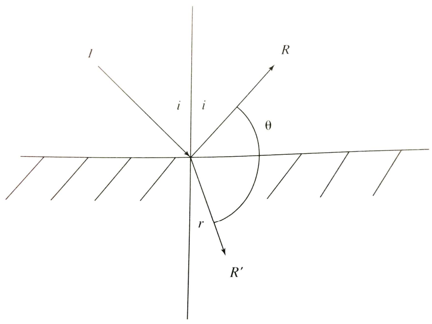

FIGURE 6.10. Light with intensity I incident on an interface between two dielectric media is split into a reflected wave R and a refracted wave R'. The angles of incidence and refraction r are shown.

Example 6.7

- Show the Brewster's law describes the condition that θ in Figure 6.10 is 90°, and find Brewster's angle for glass with n=1.612.

Example 6.8

- What is Brewster's angle for diamond with *n*=2.42?

Figure 6.11

- Produce polarized light using a stack of glass plates set at the Brewster angle.
- Each successive plate removes light polarized parallel to the surface of the plates, i.e. normal to the plane of the figure.
- With a sufficient number of plates, the emerging wave is polarized in the plane of the figure.

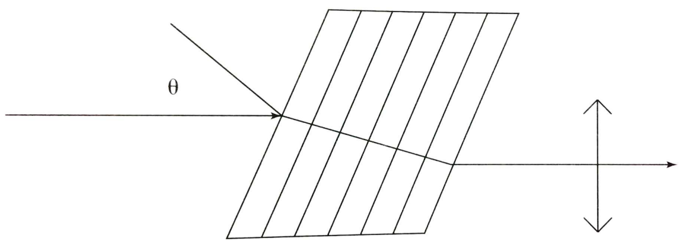

FIGURE 6.11. Polarization using the Brewster angle with a stack of transparent plates.

Homework

- 5, 6, 8, 11, 14, 15